

Direct CP Violation in D Decays in view of LHCb and CDF Results ¹

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The LHCb and CDF Collaborations have recently reported evidence for a CP asymmetry around -0.7% in ΔA_{CP} , the difference between $A_{CP}(D^0 \rightarrow K^+ K^-)$ and $A_{CP}(D^0 \rightarrow \pi^+ \pi^-)$. In the Standard Model this effect may be accounted for by enhanced $1/m_c$ corrections in a CP-violating penguin amplitude governed by a Cabibbo-Kobayashi-Maskawa (CKM) factor $V_{cb}^* V_{ub}$. A consistent scheme based on broken flavor SU(3) is presented relating Cabibbo-favored (CF) and singly-Cabibbo-suppressed (SCS) D meson decay rates into two pseudoscalars. Two important ingredients supporting the above interpretation for ΔA_{CP} are a large exchange amplitude in CF decays which is formally $1/m_c$ -suppressed, and a pure $\Delta U = 0$ U-spin breaking CP-conserving penguin amplitude involving $V_{cs}^* V_{us}$ and $V_{cd}^* V_{ud}$ which accounts for the difference between the $D^0 \rightarrow K^+ K^-$ and $D^0 \rightarrow \pi^+ \pi^-$ decay rates. The magnitudes of the CP conserving and CP violating penguin amplitudes, where the former involves U-spin breaking at a level of 10%, are shown to be related to each other by the magnitudes

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of corresponding CKM factors. This simple scheme leads to preferable sign predictions for CP asymmetries in charmed meson decays into two pseudoscalars and to correlations between asymmetries in two pairs of these processes.

1 Introduction

CP violation was first observed at a level of 10^{-3} in kaon decays. A variety of CP asymmetries have been measured subsequently at the tens of percent level in B decays [1, 2]. The Standard Model, which accounts adequately for all these unique observations [3], has often been used to claim predicting naturally very small CP asymmetries in SCS decays of charmed particles, of order 10^{-3} or less [4, 5, 6, 7]. These decays, dominated by physics of the first two quark families, involve a tiny CP-violating penguin contribution of the third family (P_b) suppressed both by the smallness of the CKM matrix element $V_{cb}^*V_{ub}$ and by the relatively small b quark mass (compared to a large penguin amplitude in B decays involving $V_{tb}^*V_{ts}$ and the heavy t quark mass). However, a very early study has proposed that the penguin amplitude in SCS D decays may be enhanced by nonperturbative effects in analogy to the $s \rightarrow d$ penguin amplitude in $K \rightarrow \pi\pi$ [8].

During the first half of 2012 the LHCb and CDF Collaborations have reported independently evidence for CP-violating charm decays in the difference ΔA_{CP} between CP asymmetries in $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$. The averaged value of the two asymmetry measurements was found to be nonzero at $\sim 4\sigma$ at the somewhat unexpected high level of about $\Delta A_{CP} \simeq -0.7\%$ [9, 10, 11]. These results have led to a continuous flood of theory papers, some trying to recover these asymmetries within the Standard Model [12]–[21], while others offering also interpretations beyond the Standard Model [12, 13], [22]–[32].

Motivated by the LHCb and CDF experimental results described in Sec. 2, we will start this review by discussing in Sec. 3 a crude estimate of ΔA_{CP} within the Standard Model. We point out an intrinsic uncertainty in the estimated asymmetry from $1/m_c$ corrections, presenting in Sec. 4 experimental evidence for such large corrections in decay rates for CF D decays. In Sec. 5 we describe a scheme [17] based on broken flavor SU(3) relating decay rates for CF and SCS charmed meson decays, which accounts consistently and quite precisely for both CF and SCS decay rates of D mesons into two pseudoscalars, explaining in particular the different decay rates for $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$. We introduce two types of U-spin breaking in SCS decays: (1) U-spin breaking in $\Delta U = 1$ tree amplitudes given in terms of ratios of light pseudoscalar meson decay constants and ratios of D meson form factors. (2) A pure U-spin breaking $\Delta U = 0$ penguin amplitude. In a short Sec. 6 we show that a 10% U-spin breaking in this penguin amplitude, required to account for SCS decay rates, can explain naturally the measured CP asymmetry ΔA_{CP} in terms of

a penguin amplitude P_b involving $V_{cb}^*V_{ub}$. In this framework all CP asymmetries in SCS D decays into two pseudocalars depend on two parameters, the magnitude and strong phase of P_b . Consequently one expects correlations between asymmetries in different processes. We discuss such correlations in Sec. 7 and conclude in Sec. 8.

2 LHCb and CDF asymmetry measurements

One defines a CP asymmetry in $D \rightarrow f$:

$$A_{CP}(f) \equiv \frac{\Gamma(D \rightarrow f) - \Gamma(\overline{D} \rightarrow \overline{f})}{\Gamma(D \rightarrow f) + \Gamma(\overline{D} \rightarrow \overline{f})} . \quad (1)$$

For a CP-eigenstate, such as $f = \pi^+\pi^-$ or K^+K^- ,

$$\begin{aligned} A_{CP}(f_{CP}) &\equiv \frac{\Gamma(D^0 \rightarrow f_{CP}) - \Gamma(\overline{D}^0 \rightarrow f_{CP})}{\Gamma(D^0 \rightarrow f_{CP}) + \Gamma(\overline{D}^0 \rightarrow f_{CP})} \\ &\approx A_{CP}^{\text{dir}}(f) + \frac{\langle t(f) \rangle}{\tau_D} A_{CP}^{\text{ind}}(f) , \end{aligned} \quad (2)$$

where t is the proper decay time and τ_D is the D meson lifetime.

The indirect asymmetry A_{CP}^{ind} induced by D^0 - \overline{D}^0 mixing (q/p) is approximately

$$A_{CP}^{\text{ind}}(f) \sim x \sin \phi \quad (x \equiv \frac{\Delta m}{\Gamma}), \quad \phi \equiv \arg \left[(q/p)(\overline{A}_f/A_f) \right] . \quad (3)$$

Thus $A_{CP}^{\text{ind}}(f)$ is approximately independent of f because A_f and \overline{A}_f involves a tiny weak phase difference.

One now defines the difference of two asymmetries:

$$\Delta A_{CP} \equiv A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) \approx \Delta A_{CP}^{\text{dir}} + \frac{\Delta \langle t \rangle}{\tau_D} A_{CP}^{\text{ind}} , \quad (4)$$

where

$$\Delta \langle t \rangle \equiv \langle t(K^+K^-) \rangle - \langle t(\pi^+\pi^-) \rangle . \quad (5)$$

The measurements of LHCb and CDF are:

LHCb [9]

$$\begin{aligned} \Delta A_{CP} &= [-0.82 \pm 0.21(\text{stat}) \pm 0.11(\text{syst})]\% , \\ \frac{\Delta \langle t \rangle}{\tau_D} &= [9.83 \pm 0.22(\text{stat}) \pm 0.19(\text{syst})]\% , \end{aligned} \quad (6)$$

CDF [10]

$$\begin{aligned}\Delta A_{CP} &= [-0.62 \pm 0.21(\text{stat}) \pm 0.10(\text{syst})]\% , \\ \frac{\Delta\langle t \rangle}{\tau_D} &= [27 \pm 1]\% .\end{aligned}\tag{7}$$

Combining the two results and assuming uncorrelated Gaussian errors, one finds averages [33]:

$$\Delta A_{CP}^{\text{dir}} = (-0.656 \pm 0.154)\% , \quad A_{CP}^{\text{ind}} = (-0.025 \pm 0.231)\% .\tag{8}$$

Thus the direct asymmetry differs from zero by about 4σ , while the indirect asymmetry is consistent with zero. We note that earlier asymmetry measurements by the E687 [34], E791 [35], FOCUS [36], CLEO [37], BaBar [38] and Belle [39] collaborations have a negligible effect on these averages.

We will also use results obtained by CDF for separate asymmetries in $D^0 \rightarrow K^+ K^-$ and $D^0 \rightarrow \pi^+ \pi^-$ [40],

$$\begin{aligned}A_{CP}(D^0 \rightarrow K^+ K^-) &= (-0.24 \pm 0.22 \pm 0.09)\% , \\ A_{CP}(D^0 \rightarrow \pi^+ \pi^-) &= (0.22 \pm 0.24 \pm 0.11)\% ,\end{aligned}\tag{9}$$

for which we calculate the corresponding 90% confidence level limits:

$$-0.63\% \leq A_{CP}(D^0 \rightarrow K^+ K^-) \leq 0.15\% , \quad -0.21\% \leq A_{CP}(D^0 \rightarrow \pi^+ \pi^-) \leq 0.65\% .\tag{10}$$

3 Estimate of ΔA_{CP} in the Standard Model

SCS charmed meson decays are described by an effective weak Hamiltonian H_{eff} at a scale $\mu \sim m_c$:

$$H_{\text{eff}}^{\text{SCS}} = \frac{G_F}{\sqrt{2}} \left[\Sigma_{i=d,s} V_{ci}^* V_{ui} (C_1 Q_1^i + C_2 Q_2^i) - V_{cb}^* V_{ub} \Sigma_{j=3}^6 C_j Q_j + C_{8g} Q_{8g} \right] + h.c. \tag{11}$$

The $(V - A) \times (V - A)$ current-current “tree” operators $Q_{1,2}^i$ have flavor structure $(\bar{i}c)(\bar{u}i)$ where $i = d, s$, while the penguin operators $Q_{3,\dots,6}$ have a structure $(\bar{u}c)\Sigma_q(\bar{q}q)$, where $q = u, d, s$. The Wilson coefficients can be evaluated in perturbation theory at $\mu \sim m_c$. In contrast, it is extremely difficult to calculate the hadronic matrix elements of these operators and of the chromomagnetic operator $Q_{8g} \equiv -\frac{g_s}{8\pi^2} m_c \bar{u} \sigma_{\mu\nu} (1 + \gamma_5) G^{\mu\nu} c$.

An estimate of the dominant contribution to ΔA_{CP} , which originates in the interference of tree and penguin amplitudes, requires calculating the ratio $r \equiv |P_b|/|T|$

of penguin and tree amplitudes for $D^0 \rightarrow \pi^+\pi^-$ and $D^0 \rightarrow K^+K^-$. Denoting by $\delta_{\pi\pi}$ the strong phase difference between these amplitudes in $D^0 \rightarrow \pi^+\pi^-$, one has

$$A_{CP}(D^0 \rightarrow \pi^+\pi^-) \approx 2r \sin \delta_{\pi\pi} \sin \gamma . \quad (12)$$

As we will show in Sec. 5, the asymmetries in $D^0 \rightarrow \pi^+\pi^-$ and $D^0 \rightarrow K^+K^-$ are related by an approximate U-spin symmetry,

$$A_{CP}(D^0 \rightarrow K^+K^-) \simeq -A_{CP}(D^0 \rightarrow \pi^+\pi^-) , \quad (13)$$

thus

$$\Delta A_{CP} \simeq -4r \sin \delta_{\pi\pi} \sin \gamma . \quad (14)$$

The weak phase γ is known to be around 70° [41] with $\sin \gamma \simeq 1$. One may assume for simplicity $\sin \delta_{\pi\pi} \sim 1$, hence $\Delta A_{CP} \sim -4r$. The LHCb and CDF averaged direct asymmetry in Eq. (8) correspond to $r \sim (17 \pm 4) \times 10^{-4}$. Factoring out CKM matrix elements in P_b and T by defining $P_b \equiv V_{cb}^* V_{ub} \mathcal{P}_b$ and $T = V_{cd(s)}^* V_{ud(s)} \mathcal{T}$, one has [42]:

$$r \equiv \frac{|P_b|}{|T|} = \frac{|V_{cb}^* V_{ub}|}{|V_{cd}^* V_{ud}|} \frac{|\mathcal{P}_b|}{|\mathcal{T}|} \simeq 7 \times 10^{-4} \frac{|\mathcal{P}_b|}{|\mathcal{T}|} \quad \text{implying} \quad \frac{|\mathcal{P}_b|}{|\mathcal{T}|} \sim 2 - 3 . \quad (15)$$

The question is: are we able to provide a reliable estimate for $|\mathcal{P}_b|/|\mathcal{T}|$, the ratio of reduced penguin and tree matrix elements?

A naive estimate based on perturbation theory would suggest $|\mathcal{P}_b|/|\mathcal{T}| \sim \alpha_s(m_c)/\pi$, lying an order of magnitude below (15). However, as m_c is not much larger than Λ_{QCD} , one expects large $1/m_c$ corrections to this leading order estimate. The most direct evidence for such large $1/m_c$ corrections is provided by a large exchange amplitude contributing to CF D decay rates to be discussed in the next section.

Detailed calculations of $1/m_c$ corrections in r are quite challenging and involve subtle sizable uncertainties. Ref. [14] shows, for instance, that insertion of penguin operators into power-suppressed annihilation amplitudes, accompanied by penguin contraction matrix elements of tree operators (which are also formally $1/m_c$ suppressed) are enhanced relative to the above leading order estimate. Thus Ref. [14] concludes that the measured asymmetry ΔA_{CP} can be accommodated within the Standard Model.

Other recent attempts for calculating r using somewhat different approaches demonstrate the difficulty of this calculation. Ref. [18] assumes factorization for parametrizing nonperturbative hadronic matrix elements of penguin operators, thus obtaining an asymmetry considerably smaller than measured. Ref. [19] applies an oversimplified two (or three) channel coupled S matrix for $\pi\pi$ and KK final states, assuming SU(3) breaking of order one in $D^0 \rightarrow K^0 \bar{K}^0$ (in contrast, we show that SU(3) breaking of order 10% in an enhanced CP-conserving penguin amplitude suffices), thereby obtaining an asymmetry which is marginally consistent with the ΔA_{CP} measurement.

Decay	Amplitude	Theoretical _%	Experimental _% [42]
$D^0 \rightarrow K^- \pi^+$	$T + E$	3.90	3.88 ± 0.05
$D^0 \rightarrow \bar{K}^0 \pi^0$	$(C - E)/\sqrt{2}$	2.35	2.38 ± 0.08
$D^0 \rightarrow \bar{K}^0 \eta$	$C/\sqrt{3}$	1.00	0.96 ± 0.06
$D^0 \rightarrow \bar{K}^0 \eta'$	$-(C + 3E)/\sqrt{6}$	1.92	1.88 ± 0.10
$D^+ \rightarrow \bar{K}^0 \pi^+$	$T + C$	3.09	2.93 ± 0.09
$D_s^+ \rightarrow \bar{K}^0 K^+$	$C + A$	2.94	2.96 ± 0.16
$D_s^+ \rightarrow \pi^+ \eta$	$(T - 2A)/\sqrt{3}$	1.81	1.83 ± 0.15
$D_s^+ \rightarrow \pi^+ \eta'$	$2(T + A)/\sqrt{6}$	3.60	3.94 ± 0.33

Table 1: Amplitude representations and comparison of experimental and fit branching ratios for CF decays of charmed mesons to two pseudoscalars.

Thus we conclude conservatively that the seemingly large value measured for ΔA_{CP} can be accounted for by penguin enhancement and is not inconsistent with the Standard Model. This situation, anticipated many years ago in Ref. [8], resembles the situation of the observed $\Delta I = 1/2$ enhancement in $K \rightarrow \pi\pi$, which originates to a large extent from penguin dominance [43]. Both effects involve nonperturbative uncertainties.

In the next two sections we show evidence both for large $1/m_c$ -suppressed exchange amplitudes contributing to CF decay rates, and for enhanced CP-conserving penguin amplitudes contributing to SCS D decay rates. Subsequently we present in this scheme CP asymmetry predictions for a whole class of SCS nonstrange charmed meson decays into two light pseudoscalars.

4 CF D decays into two pseudoscalars

CF D meson decays into two pseudoscalars are conveniently described in terms of four types of SU(3) flavor-topology matrix elements of tree operators, T (color-favored), C (color-suppressed), E (exchange), and A (annihilation) [44, 45, 46, 47]. This description, which relates amplitudes of CF decays to those of SCS decays (see next section), is equivalent to group theoretical expressions in terms of SU(3) reduced amplitudes. (See [8, 15] and references therein.) A fit to decay rates performed in Ref. [46] using the kinematic relation between decay amplitudes and decay rates, $\Gamma = |A|^2 p_{\text{cm}}/8\pi M_D^2$, leads to results given in Table 1. The rather good quality fit, involving a slight dependence on η - η' mixing (the values in Table 1 correspond to $\theta_\eta = 19.5^\circ$), obtains the following complex amplitudes in units of 10^{-6} GeV:

$$T = 2.93, \quad C = 2.34 e^{-i152^\circ}, \quad E = 1.57 e^{i121^\circ}, \quad A = 0.33 e^{i170^\circ}. \quad (16)$$

Five comments are in order when considering CF amplitudes:

- The amplitude E is formally suppressed by $1/m_c$ relative to T . The experimental ratio $|E|/|T| > 0.5$ implies large $1/m_c$ corrections in E . This point has been alluded to in the preceding section, providing a basis for expecting large $1/m_c$ corrections also in SCS decays.
- The four topological amplitudes involve large relative strong phases. These phases originate in final state interactions which are at least partially due to nearby resonances [48].
- CF decays involve a single CKM factor $V_{cs}^*V_{ud}$ and obtain no contribution from a penguin amplitude. Thus one expects $A_{CP} = 0$ in these decays.
- The effective weak Hamiltonian inducing CF decays has flavor structure $(\bar{s}c)(\bar{u}d)$ and is $\Delta U = 1$. U-spin is an SU(2) subgroup of flavor SU(3), under which (d, s) transforms like a doublet while u and c are singlets.
- CF and doubly-Cabibbo-suppressed D decays (not discussed in this review) are related to each other by a U-spin transformation, $d \leftrightarrow s$ [49].

5 SCS D decays into two pseudoscalars

The effective weak Hamiltonian for SCS decays involves current-current (tree) operators with flavor structure

$$H_{\text{tree}}^{\text{SCS}} \propto \lambda [(\bar{s}c)(\bar{u}s) - (\bar{d}c)(\bar{u}d)] , \quad (17)$$

where $\lambda \equiv \tan \theta_{\text{Cabibbo}} = 0.2317$. We have neglected a small weak phase difference between $V_{cs}^*V_{us}$ and $-V_{cd}^*V_{ud}$. In fact, unitarity of the CKM matrix

$$V_{cd}^*V_{ud} + V_{cs}^*V_{us} + V_{cb}^*V_{ub} = 0 \quad \text{implies} \quad \text{Arg} \left(\frac{V_{cs}^*V_{us}}{-V_{cd}^*V_{ud}} \right) \approx \frac{|V_{cb}| |V_{ub}|}{|V_{cd}| |V_{ud}|} \sin \gamma \simeq 7 \times 10^{-4} . \quad (18)$$

We will argue in Sec. 6 that the effect of this tiny phase on CP asymmetries in SCS decays is negligible.

Within flavor SU(3) the amplitudes contributing to SCS processes are the four tree amplitudes in CF decays, T, C, E and A multiplied by $V_{cs}^*V_{us}/V_{cs}^*V_{ud} = \lambda$ or $V_{cd}^*V_{ud}/V_{cs}^*V_{ud} = -\lambda$. The operator $H_{\text{tree}}^{\text{SCS}}$ transforms like $\Delta U = 1$; hence in the U-spin symmetry limit (using the transformation $d \leftrightarrow s$) one has

$$A_{\text{tree}}(D^0 \rightarrow K^+ K^-) = -A_{\text{tree}}(D^0 \rightarrow \pi^+ \pi^-) = \lambda(T + E) . \quad (19)$$

The tiny CP-violating penguin operator in Eq. (11), of flavor structure $(\bar{u}c)\Sigma_q(\bar{q}q)$, is a U-spin singlet. Therefore in the U-spin symmetry limit

$$A_{\text{penguin}}(D^0 \rightarrow K^+ K^-) = A_{\text{penguin}}(D^0 \rightarrow \pi^+ \pi^-) , \quad (20)$$

implying

$$A_{CP}(D^0 \rightarrow K^+ K^-) = -A_{CP}(D^0 \rightarrow \pi^+ \pi^-) , \quad (21)$$

as has already been mentioned in Eq. (13).

We now discuss U-spin symmetry breaking which plays an important role in SCS decays. The two tree amplitudes measured for $D^0 \rightarrow \pi^+ \pi^-$ and $D^0 \rightarrow K^+ K^-$, which are equal in the U spin symmetry limit, seem to be experimentally quite different in magnitude (we neglect the tiny penguin contribution P_b) [42],

$$\frac{|A(D^0 \rightarrow K^+ K^-)|}{|A(D^0 \rightarrow \pi^+ \pi^-)|} = 1.81 \pm 0.04 . \quad (22)$$

Typical U-spin breaking [or SU(3) breaking], such as in f_K/f_π or in $F(D \rightarrow K)/F(D \rightarrow \pi)$, is at most of order 0.2 – 0.3. The ratio (22) may combine several sources of U-spin breaking. U-spin breaking in factorized (color-favored) tree amplitudes [50, 51] is given in terms of a product of ratios of light meson decay constants and D meson form factors [52, 53] (neglecting the contribution of $f_-(q^2)$ at $q^2 = m_{\pi,K}^2$):

$$\frac{f_K}{f_\pi} \frac{f_+(D \rightarrow K)(m_K^2)}{f_+(D \rightarrow \pi)(m_\pi^2)} = 1.38 . \quad (23)$$

This factor is expected to affect the ratio $|A_{\text{tree}}(D^0 \rightarrow K^+ K^-)|/|A_{\text{tree}}(D^0 \rightarrow \pi^+ \pi^-)|$ through the contributions of T but not of E . [See Eq. (19).] Thus the ratio of $\Delta U = 1$ tree contributions is smaller than 1.38 and is insufficient by itself to account for the measured ratio of amplitudes (22).

This situation requires another contribution to U-spin breaking from $\Delta U = 0$ operators. Indeed, using flavor topology amplitudes such a contribution is naturally provided by $c \rightarrow u$ penguin amplitudes involving d and s quarks with different masses in the intermediate state [17]. We note that these amplitudes involve the same CKM factor λ as the $\Delta U = 1$ tree amplitudes, thereby affecting decay rates without leading to CP asymmetries. (As mentioned, we neglect a tiny phase difference between $V_{cs}^* V_{us}$ and $-V_{cd}^* V_{ud}$; its negligible effect on CP asymmetries will be discussed in the next section.) These penguin amplitudes are expected naturally to increase $|A(D^0 \rightarrow K^+ K^-)|$ and decrease $|A(D^0 \rightarrow \pi^+ \pi^-)|$ as required, because they contribute equally with equal signs to these amplitudes, while tree amplitudes contributions involve opposite signs. [See Eq. (19).]

We denote these pure U-spin breaking $\Delta U = 0$ contributions by $P \equiv P_s - P_d$ and $PA \equiv PA_s - PA_d$, representing differences between s and d contributions from penguin and penguin annihilation topologies. In order to include SU(3) breaking in tree amplitudes we define

$$T_\pi = T \cdot \frac{|f_{+(D^0 \rightarrow \pi^-)}(m_\pi^2)|}{|f_{+(D^0 \rightarrow K^-)}(m_\pi^2)|} \cdot \frac{m_D^2 - m_\pi^2}{m_D^2 - m_K^2}, \quad (24)$$

$$T_K = T \cdot \frac{|f_{+(D^0 \rightarrow K^-)}(m_K^2)|}{|f_{+(D^0 \rightarrow K^-)}(m_\pi^2)|} \cdot \frac{f_K}{f_\pi}. \quad (25)$$

Decay Mode	Amplitude representation A_f	$ A_f $ (10^{-7} GeV)		ϕ_T^f degrees
		Experiment	Theory	
$D^0 \rightarrow \pi^+\pi^-$	$-\lambda(T_\pi + E) + (P + PA)$	4.70 ± 0.08	4.70	-158.5
$D^0 \rightarrow K^+K^-$	$\lambda(T_K + E) + (P + PA)$	8.49 ± 0.10	8.48	32.5
$D^0 \rightarrow \pi^0\pi^0$	$-\lambda(C - E)/\sqrt{2} - (P + PA)/\sqrt{2}$	3.51 ± 0.11	3.51	60.0
$D^0 \rightarrow K^0\bar{K}^0$	$-(P + PA) + P$	2.39 ± 0.14	2.37	-145.6
$D^+ \rightarrow \pi^+\pi^0$	$-\lambda(T_\pi + C)/\sqrt{2}$	2.66 ± 0.07	2.26	126.3
$D^+ \rightarrow K^+\bar{K}^0$	$\lambda(T_K - A) + P$	6.55 ± 0.12	6.87	-4.2

Table 2: Amplitude representations quoting strong phases; comparison of experimental and fit branching ratios for SCS decays of charmed mesons to two pseudoscalars.

Amplitude representations A_f for six SCS decays of nonstrange charmed mesons into two pseudoscalars, and strong phases ϕ_T^f of these amplitudes calculated with respect to T , are listed in Table 2. Input values for tree amplitudes, T, C, E and A involving strong phases were obtained in Eq. (16) using CF decay rates. The experimental decay rates (or the quoted measured magnitudes of amplitudes A_f) for the first three SCS processes in Table 2 are used to fit the complex amplitude $P + PA$.

The extracted value [17],

$$P + PA = (0.44 + 1.41i) (10^{-7} \text{ GeV}), \text{ with magnitude } |P + PA| = 1.48 \times (10^{-7} \text{ GeV}), \quad (26)$$

is seen to fit the data excellently, corresponding to a very low χ^2 for a single degree of freedom. In particular, using CF decay rates as input, this U-spin-breaking amplitude accounts very well for the quite different decay rates of $D^0 \rightarrow \pi^+\pi^-$ and $D^0 \rightarrow K^+K^-$, which seemed *ab initio* like a puzzle. It also fits well the measured decay rate for $D^0 \rightarrow \pi^0\pi^0$.

We note in passing that the ratio of tree contributions to the two amplitudes for $D^0 \rightarrow \pi^+\pi^-$ and $D^0 \rightarrow K^+K^-$ [17],

$$\begin{aligned} |A_{\text{tree}}(D^0 \rightarrow \pi^+\pi^-)| &= \lambda|T_\pi + E| = 5.73 \times (10^{-7} \text{ GeV}), \\ |A_{\text{tree}}(D^0 \rightarrow K^+K^-)| &= \lambda|T_K + E| = 7.42 \times (10^{-7} \text{ GeV}), \end{aligned} \quad (27)$$

involves U-spin breaking of 29% which, as anticipated, is indeed smaller than U-spin breaking in the ratio (23).

The approximately real extracted value of P , $P = (-1.52 + 0.08i) (10^{-7} \text{ GeV})$, comparable in magnitude to $|P + PA|$ but involving a large error, is obtained using rate measurements for $D^0 \rightarrow K^0\bar{K}^0$ and $D^+ \rightarrow K^+\bar{K}^0$ [17]. We note that while all four D^0 decay rates are fitted excellently, the fit to the two D^+ decay rates is of only moderate quality.

6 Estimating U-spin breaking in $P + PA$ from P_b

Both the CP-conserving penguin amplitude, $P + PA$ or P (which are of comparable magnitudes and are purely U-spin breaking), and the CP-violating penguin amplitudes P_b are of nonperturbative nature. Assuming that their enhancement has a common origin due to $1/m_c$ corrections, we will estimate the level of U-spin breaking in $P + PA$. (See also discussion in Ref. [20].)

As mentioned, these two amplitudes involve different CKM factors, $V_{cs}^*V_{us}$ (or $V_{cd}^*V_{ud}$) and $V_{cb}^*V_{ub}$, respectively, which may be factored out defining $P + PA \equiv V_{cs}^*V_{us}(\mathcal{P} + \mathcal{PA})$ and $P_b \equiv V_{cb}^*V_{ub}\mathcal{P}_b$. [See also line above Eq. (15)]. The ratio

$$|\mathcal{P} + \mathcal{PA}|/|\mathcal{T}| = |P + PA|/|T| = 0.20 - 0.26 , \quad (28)$$

is calculated from Eq. (26) and (27), where T is the tree amplitude in $D^0 \rightarrow \pi^+\pi^-$ or $D^0 \rightarrow K^+K^-$. (These two tree amplitudes differ by about 30%.) A ratio $|\mathcal{P}_b|/|\mathcal{T}| = 2 - 3$ was calculated in Eq. (15) using the average value of ΔA_{CP} measured by LHCb and CDF and assuming a large strong phase difference δ between \mathcal{P}_b and \mathcal{T} . Taking the ratio of these two ratios we conclude:

$$\frac{|\mathcal{P} + \mathcal{PA}|}{|\mathcal{P}_b|} \simeq 0.10 . \quad (29)$$

This value provides an estimate for U-spin breaking in $P + PA$,

$$\frac{|(P_s + PA_s) - (P_d + PA_d)|}{|P_s + PA_s|} \sim 0.10 , \quad (30)$$

which seems quite reasonable.

We now discuss briefly the effect on CP asymmetries in SCS decays of a small weak phase difference between $V_{cs}^*V_{us}$ and $-V_{cd}^*V_{ud}$ ($\simeq 7 \times 10^{-4}$) which has been neglected in Eq. (17). Using unitarity of the CKM matrix one may rewrite P and PA in an explicitly U-spin breaking form,

$$P = V_{cs}^*V_{us}(\mathcal{P}_s - \mathcal{P}_d) , \quad PA = V_{cs}^*V_{us}(\mathcal{PA}_s - \mathcal{PA}_d) , \quad (31)$$

with $P_b = V_{cb}^*V_{ub}(\mathcal{P}_b - \mathcal{P}_d)$, $PA_b = V_{cb}^*V_{ub}(\mathcal{PA}_b - \mathcal{PA}_d)$. In the next section we will assume that PA_b is negligible relative to P_b , proposing tests for this assumption.

Interference of tree and $P + PA$ amplitudes in $D^0 \rightarrow K^+K^-$ does not contribute to the asymmetry in this process because the two amplitudes involve a common CKM factor $V_{cs}^*V_{us}$ and a common weak phase. On the other hand, interference of corresponding amplitudes in $D^0 \rightarrow \pi^+\pi^-$ involves the above small phase difference, thus leading to a nonzero contribution to $A_{CP}(D^0 \rightarrow \pi^+\pi^-)$. Using information about magnitudes and strong phases of these amplitudes, we calculate this contribution to be at a negligible level of 1×10^{-4} . A contribution at the same level is calculated for $A_{CP}(D^+ \rightarrow K^+\bar{K}^0)$ from interference of T_K and A amplitudes also involving this tiny weak phase difference.

Decay Mode	Amplitude representation
$D^0 \rightarrow \pi^+ \pi^-$	$-\lambda (T_\pi + E) + (P + PA) + P_b$
$D^0 \rightarrow K^+ K^-$	$\lambda (T_K + E) + (P + PA) + P_b$
$D^0 \rightarrow \pi^0 \pi^0$	$-\lambda (C - E)/\sqrt{2} - (P + PA)/\sqrt{2} - P_b/\sqrt{2}$
$D^0 \rightarrow K^0 \bar{K}^0$	$-(P + PA) + P$
$D^+ \rightarrow \pi^+ \pi^0$	$-\lambda (T_\pi + C)/\sqrt{2}$
$D^+ \rightarrow K^+ \bar{K}^0$	$\lambda (T_K - A) + P + P_b$

Table 3: Amplitude representations including P_b for SCS decays of charmed mesons to two pseudoscalars.

7 Predicting CP asymmetries in SCS decays

Assuming flavor SU(3) for the CP-violating amplitude P_b [SU(3) breaking in the corresponding CP-conserving amplitude $P + PA$ has been shown to be only about 10%], we now include P_b in the six amplitudes in Table 2. The resulting complete amplitude representations are given in Table 3.

Denoting $P_b \equiv |P_b| e^{i(\delta-\gamma)}$ the amplitude for $D \rightarrow f$ may be written as

$$\mathcal{A}(D \rightarrow f) = |A_f| e^{i\phi_T^f} \left(1 + \frac{|P_b|}{|A_f|} e^{i(\delta-\phi_T^f-\gamma)} \right), \quad (32)$$

where $|A_f|$ and ϕ_T^f were defined and calculated in Table 2. The amplitude for $\bar{D} \rightarrow \bar{f}$ is obtained from this expression by $-\gamma \rightarrow \gamma$. Using the definition (1) one obtains CP asymmetries

$$\begin{aligned} A_{CP}(f) &= \frac{2(|P_b|/|A_f|) \sin \gamma \sin(\delta - \phi_T^f)}{1 + (|P_b|/|A_f|)^2 + 2(P_b/|A_f|) \cos \gamma \cos(\delta - \phi_T^f)} \\ &\approx 2(|P_b|/|A_f|) \sin \gamma \sin(\delta - \phi_T^f), \end{aligned} \quad (33)$$

where terms quadratic in $|P_b|/|A_f|$ have been neglected in the second line.

Thus, given the values for $|A_f|, \phi_T^f$ in Table 2 and $\gamma \simeq 70^\circ$ [41], all asymmetries depend on two unknown parameters, $|P_b|$ and δ . Two of the asymmetries, for $D^+ \rightarrow \pi^+ \pi^0$ and $D^0 \rightarrow K^0 \bar{K}^0$, vanish because of the absence of a P_b contribution in these processes. We will discuss these special cases below. We note that the phases ϕ_T^f have a common sign ambiguity, $\phi_T^f \rightarrow -\phi_T^f$, and the CP asymmetries (33) are approximately invariant under a joint transformation $\phi_T^f \rightarrow -\phi_T^f$, $\delta \rightarrow \pi - \delta$.

The averaged asymmetry difference ΔA_{CP} (8) measured by LHCb and CDF may be used to constrain $|P_b|$ and δ . The allowed values of $p \equiv |P_b|$ are plotted in Fig. 1 as a function of δ for a range of δ , $-2.64 \leq \delta \leq 0.41$, consistent with (10). For a wide range of δ a CP-violating penguin amplitude of magnitude $|P_b| = 0.01 \times (10^{-7} \text{ GeV})$

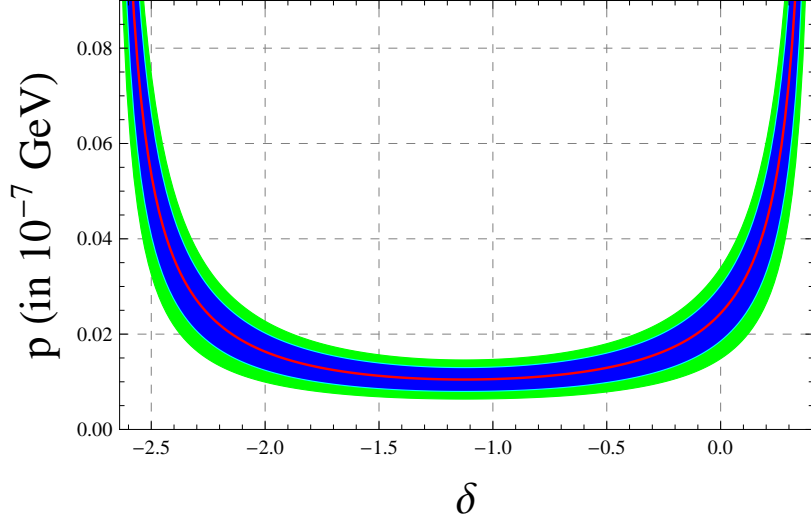


Figure 1: $p \equiv |P_b|$ and δ allowed by the measured value of ΔA_{CP} . The (red) line represents central value, while inner (blue) and outer (green) bands respectively represent 68% confidence level (1σ) and 90% confidence level (1.64σ) regions based on error in ΔA_{CP} .

is sufficient to account for the observed value of ΔA_{CP} . This value corresponds to $|\mathcal{P} + \mathcal{PA}|/|\mathcal{P}_b| \equiv (|P + PA|\lambda)/(|P_b|/|V_{cb}^*V_{ub}|) \simeq 0.10$, measuring U-spin breaking in $P + PA$ as already shown in (30).

In Fig 2 we plot the asymmetries for K^+K^- and $\pi^+\pi^-$ final states. We note that for most values of δ one expects

$$A_{CP}(D^0 \rightarrow K^+K^-) < 0, \quad A_{CP}(D^0 \rightarrow \pi^+\pi^-) > 0, \quad \frac{|A_{CP}(D^0 \rightarrow K^+K^-)|}{|A_{CP}(D^0 \rightarrow \pi^+\pi^-)|} < 1. \quad (34)$$

The central values (9) measured by CDF support the signs of these two asymmetries. More precise measurements of the two individual asymmetries are required for testing their signs and can help pin down the unknown strong phase δ , thereby predicting also the asymmetries in $D^0 \rightarrow \pi^0\pi^0$ and $D^+ \rightarrow K^+\bar{K}^0$.

In Fig. 3 we plot the predicted asymmetries for $D^+ \rightarrow K^+\bar{K}^0$ and $D^0 \rightarrow \pi^0\pi^0$. We note the correlation between these two asymmetries as functions of δ . The first asymmetry is negative for most values of δ , while the second asymmetry is positive except for large negative values of δ . Indeed, the central value of the current measurement [61], $A_{CP}(D^+ \rightarrow K^+\bar{K}^0) = (-0.16 \pm 0.58 \pm 0.25)\%$, is negative. The experimental error must be reduced in order to test the sign and to provide a useful constraint on δ . The property $A_{CP}(D^+ \rightarrow K^+\bar{K}^0) < 0$ is based on assuming $PA_b = 0$. This assumption can be tested through the vanishing of $A_{CP}(D^0 \rightarrow K^0\bar{K}^0)$, as the

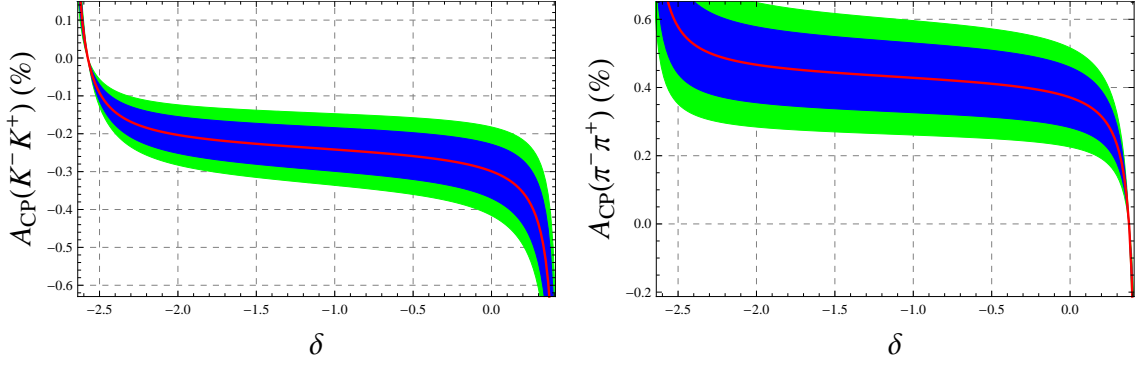


Figure 2: $A_{CP}(D^0 \rightarrow K^+ K^-)$ and $A_{CP}(D^0 \rightarrow \pi^+ \pi^-)$ as a functions of the allowed values of δ . The (red) lines represent central values, while inner (blue) and outer (green) bands respectively represent 68% confidence level (1σ) and 90% confidence level (1.64σ) regions.

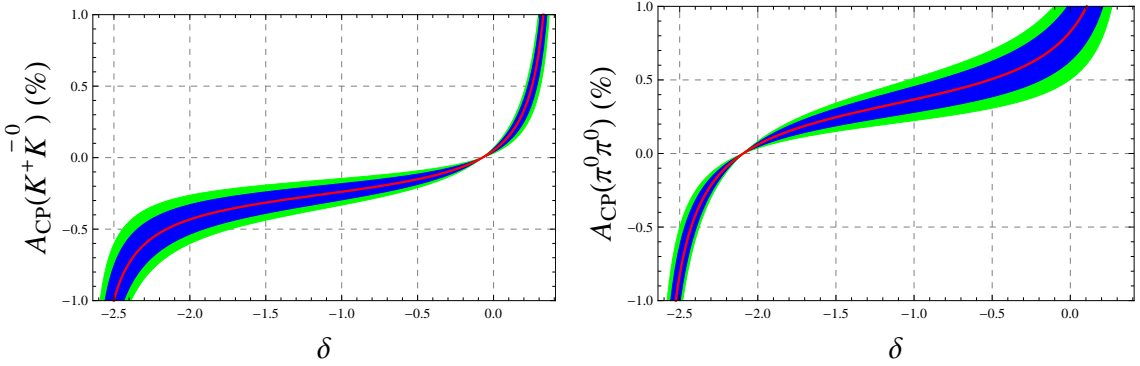


Figure 3: $A_{CP}(D^+ \rightarrow K^+ \bar{K}^0)$ and $A_{CP}(D^0 \rightarrow \pi^0 \pi^0)$ as a functions of the allowed values of δ . Color notations are as in Fig. 2.

absence of a u quark in the final state forbids a P_b contribution in this process. The latter asymmetry may obtain a small contribution from an interference of PA_b with an SU(3)-breaking term in E [21].

A quite general test of the Standard Model is $A_{CP}(D^+ \rightarrow \pi^+\pi^0) = 0$. The $\Delta I = 1/2$ amplitude P_b is absent in this process where the final state has $I = 2$. Therefore one expects $A_{CP}(D^+ \rightarrow \pi^+\pi^0)$ to vanish in the Standard Model at a very high precision. While higher order electroweak penguin amplitudes involving $\Delta I = 3/2$ are enhanced in B decays by the heavy t quark mass [55], such contributions are tiny in SCS charmed meson decays.

8 Conclusion

We conclude this talk by summarizing its main points:

- The asymmetry difference, $\Delta A_{CP} \equiv A_{CP}(D^0 \rightarrow K^+K^-) - A_{CP}(D^0 \rightarrow \pi^+\pi^-) \sim -0.7\%$, measured by the LHCb and CDF collaborations may originate in an enhancement from $1/m_c$ corrections of a CP-violating penguin amplitude P_b . These corrections which are nonperturbative are difficult to calculate reliably, reminiscent of the situation in D^0 - \bar{D}^0 mixing.
- Experimental evidence supporting the above hypothesis exists in terms of large $1/m_c$ corrections causing an enhancement of an exchange amplitude E in CF decays.
- A consistent broken flavor SU(3) framework is presented for CF and SCS decays of charmed mesons into two pseudoscalars, explaining in particular the large ratio $|A(D^0 \rightarrow K^+K^-)|/|A(D^0 \rightarrow \pi^+\pi^-)| \approx 1.8$ and accounting quite precisely for all four D^0 SCS decay rates. This scheme involves U-spin breaking in $\Delta U = 1$ tree amplitudes given by ratios of meson decay constants and D meson form factors, and a purely U-spin breaking $\Delta U = 0$ CP-conserving penguin amplitude $P + PA$.
- Normalizing the value of $|P + PA|$ fitted by decay rates by the magnitude $|P_b|$ extracted from the measured ΔA_{CP} , we find U-spin breaking in $P + PA$ to be about 10% which seems reasonable.
- This framework, which involves an unknown strong phase δ , predicts CP asymmetries of order (several) $\times 10^{-3}$ for $D^0 \rightarrow \pi^+\pi^-, K^+K^-, \pi^0\pi^0$ and $D^+ \rightarrow K^+\bar{K}^0$, with specific signs for most value of δ . Correlations as functions of δ are predicted between $A_{CP}(D^0 \rightarrow \pi^+\pi^-)$ and $A_{CP}(D^0 \rightarrow K^+K^-)$ and between $A_{CP}(D^0 \rightarrow \pi^0\pi^0)$ and $A_{CP}(D^+ \rightarrow K^+\bar{K}^0)$.

- Improving experimental limits on the individual asymmetries $A_{CP}(D^0 \rightarrow \pi^+\pi^-)$ and $A_{CP}(D^0 \rightarrow K^+K^-)$ would provide useful constraints on δ .
- Measuring a value for $A_{CP}(D^+ \rightarrow \pi^+\pi^0)$ with magnitude larger than 0.1% would provide unambiguous evidence for new physics due to $\Delta I = 3/2$ operators.

In this talk we discussed charmed meson decays into two pseudoscalar mesons. One expects asymmetries of similar magnitudes in multi-body charm decays. Several decays into a vector meson and a pseudoscalar meson, in particular with charged particles in the final state, are experimentally feasible. This includes $D^+ \rightarrow \phi(\rightarrow K^+K^-)\pi^+$, for which an asymmetry $(0.51 \pm 0.28 \pm 0.05)\%$ has been published very recently [56] and reported at this conference [57]. Such an asymmetry is due to interference of a color-suppressed tree amplitude and a flavor SU(3) singlet penguin amplitude involving $V_{cb}^*V_{ub}$ analogous to an amplitude (s_P) contributing to $B^+ \rightarrow \phi\pi^+$ [58]. Other processes which are useful and should be studied are $D^+ \rightarrow \rho^0(\rightarrow \pi^+\pi^-)\pi^+$, similar to $D^+ \rightarrow \pi^+\pi^0$ but in which CP violation is permitted by isospin, and $D^+ \rightarrow \bar{K}^{*0}(\rightarrow K^-\pi^+)K^+$ which resembles $D^+ \rightarrow \bar{K}^0 K^+$ studied in this talk.

It would also be quite interesting to study CP asymmetries in a pair of processes, $D^0 \rightarrow K_S(\rightarrow \pi^+\pi^-)K^\pm\pi^\mp$. These two asymmetries are expected to include contributions from CP violation in $K^0\text{-}\bar{K}^0$ mixing, similar to the situation in $D^+ \rightarrow K_S\pi^+$ [59, 60, 61, 62, 63].

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